

## Friction Factor Predictions - The Most Important and The Least Accurate Part of a Pipeline's Capacity Calculation

By Dr Ove Bratland

In a new book, available in digital format for free at [drbratland.com](http://drbratland.com), the author points out that most friction calculations are considerably less accurate than we tend to think. A pipe's surface roughness is in itself not sufficient to describe how the surface affects friction. Another important point made in the article is that in the Reynolds number range where high pressure gas pipelines typically operate, the friction is extremely sensitive to how the surface is treated – a strong argument for adequate internal coating in all gas pipelines.

### Introduction

When fluid flows through a pipe, friction between the pipe wall and the fluid works against the flow and is one of the most important parameters in determining a pipeline's capacity. As one would expect, many researchers have investigated it and come up with practical ways to predict such friction. It turns out that even for single-phase flow, pipe friction is a complex phenomenon, and questionable friction calculations are surprisingly common.

The now most common way to describe friction, and the one recommended by this article's author, is to use the Darcy-Weisbach friction factor  $f$ :

$$f \cong \frac{\Delta p}{l v^2} \rho \frac{d}{2}$$

Traditionally,  $f$  is found from the Moody diagram or from a relatively similar method based on the AGA-

recommendations, see Uhl (1965). To make readings from

the diagrams, the Reynold's number  $Re = \frac{vd}{\nu}$  and the

relative pipe roughness needs to be known. Those two dimensionless groups contain the average fluid velocity

$v \left[ \frac{m}{s} \right]$  one fluid property – the kinematic viscosity  $\nu \left[ \frac{m^2}{s} \right]$

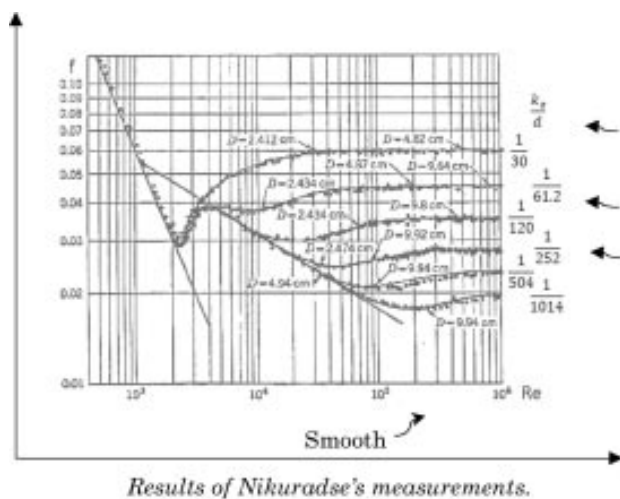
$\nu m^2$  – and two pipe properties, namely the diameter  $d [m]$  and the inner surface roughness  $k_s [m]$ .

The definition of the roughness  $k_s$  originates from Nikuradse's now more than 70 year old laboratory experiments, where he created roughness by gluing uniformly sized sand grains to the pipes' surfaces. In his experiments,  $k_s$  was simply the grains' size. Modern pipelines are of course not covered with sand, and the surface imperfections are generally not of uniform size. One very central question arises: How do we describe real surfaces, and how do we use Nikuradse's results to determine the friction in real pipes? Much of the problem with friction calculations originates in the fact that these questions have never been given good answers. The calculation methods in use today generally try to compress all relevant information about a surface into an equivalent sand grain size  $k_s$ . As one would expect, this does not always work well. The current practice works best for hydraulically smooth surfaces – the part of diagram where the roughness is small enough to be neglected for the Reynolds number of interest. It also works well for 'fully rough' part of the diagram, the part which sometimes

confusingly has been called ‘fully turbulent’ flow. But for intermediate Reynolds numbers, precisely where most high pressure gas pipelines operate, other surface properties play a significant role. Those ‘other’ properties are not well understood and not even tabulated in a way engineers can make use of. The fact is that surfaces are not only characterized by the size of their imperfections, but also by how different sorts of imperfections are distributed. It turns out that in order to get friction curves similar to the ones plotted in the Moody diagram; we need to have very low imperfection uniformity, meaning we must have a mixture of imperfections of many different sizes on the surface. If, on the other hand, our manufacturing or surface coating process creates high uniformity, we would be better off using something more similar to Nikuradse’s original diagram.

### Nikuradse’s friction factor measurements

Nikuradse (1933) was one of the most influential researchers in the field of pipe friction. He carried out a lot of measurements on relatively small pipes, and he varied  $Re$  up to  $3.4 \cdot 10^6$ . This is an order of magnitude lower than what we may encounter in gas pipelines.



### When we study Nikuradse’s curves, we see that:

- For high Reynolds numbers:** The friction factor depends only on the pipe’s relative roughness, and  $Re$  does not affect the friction factor.
- For intermediate Reynolds numbers:** In the area between laminar and ‘fully rough’ flow, the friction factor depends both on  $k_s/d$  and  $Re$ .
- For low Reynolds numbers:** The straight line in the diagram follows accepted theory, as it is also plotted in the traditional Moody diagram. The curves also show

very clearly that something happens around  $Re = 2,300$ , and this is taken as evidence that the flow no longer is fully laminar for  $Re$  higher than this value. But unlike what the Moody diagram suggests, the friction factor does not make a jump at this Reynolds number.

A much less recognized phenomenon can also be read out of the figure: For each curve except for the one with highest  $k_s/d$ , there is a minimum friction factor at one particular Reynolds number. The minima can be seen to occur for different Reynolds numbers depending on the particular pipe’s roughness. This is also something the traditional Moody diagram ignores, even though it can be important to gas pipelines.

### The traditional Moody diagram

Prandtl determined that the ‘smooth pipe’-line in Nikuradse’s diagram can be described as:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \frac{2.51}{Re\sqrt{f}}$$

Nikuradse (1933) himself presented a friction factor correlation valid for relatively high  $Re$ , a part of the diagram which sometimes confusingly has been termed ‘fully turbulent flow’, meaning the part where the curves are horizontal, as:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \frac{k_s}{3.7d}$$

The most difficult part has turned out to be describing what happens between these two extremes, in the ‘partly rough’ zone. Colebrook & White carried out additional experiments on commercial pipes in the late 1930s, and they presented what has since become the most widely used equation for estimating the friction factor in steady-state pipe flow (Colebrook, 1939). It is known as the Colebrook & White-correlation, and it has been constructed by the summarizing the two terms on the right hand side of equations 2.5.1 and 2.5.2 while keeping the left-hand side as is:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{2.51}{Re\sqrt{f}} + \frac{k_s}{3.7d} \right)$$

In spite of this equation’s popularity, we sense a problem with it immediately: It does not take the surface’s uniformity into account.

**Error! Reference source not found.** *Traditional Moody*

diagram, based on Colebrook & White's equation

### Practical friction factor calculation methods

One of the most extensive and systematic relatively recent series of measurements are those of Zagarola (1996). He has carried out measurements for  $3.1 \cdot 10^4 \leq Re \leq 3.5 \cdot 10^7$ , and he has proposed two alternative friction correlations for smooth pipes. Smooth pipe correlations are of course of great importance, given that both the traditional Moody diagram and the AGA recommendations rely on them. Any new, improved smooth-pipe correlation can therefore be used 'Moody-style' or 'AGA-Style' without changing the philosophy behind the diagrams as such.

Zagarola's two correlations give very similar results for high Reynolds numbers. The simpler of the two, which seems more than adequate considering the many uncertainties involved in normal pressure loss calculations, is:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \left( \frac{1.547}{Re \sqrt{f}} \right)^{0.9445} \right] \quad (2.9.2)$$

By multiplying out the exponents and using the analogy for how Colebrook & White combined the smooth equation with the fully rough flow, we get:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \left( \frac{1.547}{Re \sqrt{f}} \right)^{0.9445} + \frac{k_s}{3.7d} \right] \quad (2.9.3)$$

It was shown by Haaland (1983) that the sharpness in the transition from smooth to fully rough flow can be neatly adjusted by rising each of the terms in the parenthesis into a power he called  $n$ . We already explained that the more uniform the surface, meaning the more different surface imperfections show similarity with each other in shape and size, the more abrupt the transition between smooth and rough flow becomes. Utilizing this result, we define a dimensionless surface structure uniformity factor,  $u_s$ , and include it in a way similar to Haaland's mathematical exponent  $n$ , and write:

$$\frac{1}{\sqrt{f}} = -\frac{2}{u_s} \log_{10} \left[ \left( \frac{1.547}{Re \sqrt{f}} \right)^{0.9445 u_s} + \left( \frac{k_s}{3.7d} \right)^{u_s} \right]$$

The surface uniformity-based friction factor equation, equation 2.9.4 represents a clear improvement compared to Colebrook & White's correlation and the conventional Moody diagram for two reasons: It relies on improved

measurements covering a greater span of Reynolds numbers (those of Zagarola). In addition, it has a better representation of the pipe's surface roughness (using both  $k_s$  and  $u_s$ ). It also recognizes that the factor  $u_s$  is a surface property, and we have chosen to call this equation the surface uniformity-based friction factor equation. We see that unlike  $k_s$ ,  $u_s$  is dimensionless as it stands – it does not have to be divided by  $d$  or anything else.

Since  $u_s$  is a property, it can be tabulated for different types of surfaces, just like  $k_s$ . The problem is that nobody has yet done so. Until anyone has actually tabulated it for various surfaces, we cannot be sure that  $u_s$  is completely independent of  $Re$  for all different relative roughness values.

### Friction factor accuracy

So what sort of accuracy can we expect to achieve when we try to estimate the Darcy-Weisbach friction factor by using available tables, diagrams, and correlations, but without carrying out any measurements? It is an important, but difficult question. For turbulent flow, the error can be estimated by comparing the friction factor determined by equation 2.9.4 for  $u_s = 1$ , which corresponds to a Colebrook & White-like calculation (but based on improved knowledge of smooth pipe friction), and a large structure uniformity factor ( $u_s = 20$  is used here), which correspond to an AGA-like friction calculation (but also here with improved smooth pipe correlation):

$$\frac{\Delta f}{f} = \frac{f(u_s = 1) - f(u_s = 20)}{f(u_s = 1)}$$

As we can see, this error estimate is actually based on the fact that we do not know which  $u_s$  to use, since tables generally only give us  $k_s$ , not  $u_s$ .

In addition, an error for the uncertainties in the smooth-pipe curve fit, as given by Zagarola, needs to be added. He claimed that the errors could be kept within 1.2% for  $Re$  from  $3.1 \cdot 10^4$  to  $3.5 \cdot 10^7$  for his most accurate equation. By comparing this with the equation we have used, an error below 1.3 % is assumed, and this is expected to grow towards 4% for Reynolds numbers below  $3.1 \cdot 10^4$ .

It is more difficult to estimate the error in the transition zone. It requires knowledge of exactly how transition takes place. On the other hand, it is less critical to have accurate knowledge of  $k_s$  and  $u_s$  in that zone, since the curves lie closer together. For the traditional Moody diagram, the error in this area would be in the order of the difference between laminar and turbulent estimates, since even a relatively moderate error in the Reynolds number could lead to choosing the wrong correlation. That could easily lead to errors in the order of up to 100 %. The modified

Moody diagram has been designed to minimize this error, most likely down to a third or less of what the traditional Moody diagram could have produced. A combination of these considerations has been used to create the error diagram.

The results show that the friction factor error may easily become 10 or even 15 % in the 'partly rough' zone. Best agreement can be expected for 'smooth' and 'fully rough'

flow, where the error may be in the order of 2 %.

The main source of error is often going to be inaccuracy for the  $k_s$ -value selected from tabulated data (this error is not included in figure 2.11.2), not the modified Moody diagram as such. For instance, the tabulated  $k_s$  for newly painted steel is in table 2.12.2 given as "0.01-0.02 mm", the error involved in choosing  $k_s = 0,01$  mm if our paint-job

Source	Equation	Comment
Surface uniformity-based.	$Re \leq 2,300$ (laminar, correlation a): $f = \frac{64}{Re}$	Much in common with Sletfjerd's and Gersten's correlations, but interprets $u_s$ as a surface uniformity property independent of liquid properties. Also, it improves accuracy for $2,300 < Re < 20,000$ .
For $2,300 < Re \leq 3,100$ , it is based on Nikradse's measurements and Kolmogorov's turbulence theory.	$Re \geq 20,200$ (turbulent, correlation b): $\frac{1}{\sqrt{f}} = -\frac{2}{u_s} \log_{10} \left[ \left( \frac{1.547}{Re\sqrt{f}} \right)^{0.9448u_s} + \left( \frac{k_s}{3.7d} \right)^{u_s} \right]$	Diagrams in figure 2.9.1 - 2.9.3 are based on these correlations.
	$2,300 < Re \leq 3,100$ (turbulent, correlation c): Straight line from point $p_1$ to point $p_2$ , where $p_1(Re=2,300; f = \frac{64}{Re}), p_2(Re=3,100; f=0.04)$	
	$3,100 < Re \leq 20,000$ (turbulent, correlation d): Straight line from point $p_2$ to point $p_3$ , where $p_3(Re=20,000; f \text{ computed as for correlation b})$	

**Friction factor correlations to be used when no measurements available**

When measurements are available, better curve fit can normally be achieved with these equations:

Source	Equation	Comment
Surface uniformity based, suited for curve fit when measurements are available	$\frac{1}{\sqrt{f}} = -\frac{2}{u_s} \log_{10} \left[ \left( \frac{1.547}{Re\sqrt{f}} \right)^{0.9448u_s} + \left( k_d \frac{k_s}{3.7d} \right)^{u_s} \right]$ $k_d = \frac{1}{1 + e^{-\frac{Re_d - Re}{Re_d^{n_d}}}}$	One extra term added to the surface uniformity based equation, to enable it to re-produce minima,

**Friction factor correlation used for curve fitting to measurements when they exist**

is somewhat below average ( $k_s = 0.02$ , say,) can lead to a factor 2 between the correct and the used relative roughness. By looking at the modified Moody diagram, we see that this may easily lead to a 20 % error in the friction factor, depending on where in the diagram it refers to. This error, though, gets smaller and smaller the closer we come to the 'smooth flow'-line, while the errors in the error diagram have local maxima for partly rough flow. Still, it may all in all be quite common to end up with errors in the order of 20 % or more for friction factors predicted at the design stage if no relevant measurements or reliable surface data from the manufacturer is available. That may be acceptable for some applications, but it is important to be aware of the moderate accuracy. For costly pipeline projects, carrying out early tailor-made laboratory measurements may sometimes make sense.

Friction factor errors for the modified Moody diagram, mainly due to not knowing  $u_s$  accurately.

### Friction factor summary

For circular pipes carrying single-phase Newtonian fluids, friction calculations can be summarized as follows:

1. The preferred definition of the friction factor  $f$  is the Darcy-Weisbach definition, as shown in equation 2.1.6.
2. The traditional Moody diagram is still a much used way to determine  $f$ , even though some major limitations apply to it: It over-estimates the 'smooth pipe'-friction for  $Re > 10^6$ , and it is very inaccurate around  $Re = 10^4$ . Also, it neglects the fact that different pipes have different shaped friction factor curves. Therefore, the Moody diagram should be replaced by surface uniformity factor based diagrams.
3. When the surface structure uniformity factor  $u_s$  is unknown, one may opt for the most conservative estimate by setting  $u_s = 1$ . The expected friction factor accuracy can be read out of the shown friction factor diagram as a function of relative roughness and Reynolds number. Errors may be larger than 10% even when an accurate roughness value is available, and up to 20% or even more when relatively inexact, tabulated roughness values must be relied on.
4. For large pipeline projects, it may be cost-effective to improve accuracy by carrying out measurements on pipeline sections in the laboratory before making the final decision on diameter and inner surface treatment. The measurements need to focus on both  $k_s$  and  $u_s$ , meaning the friction factor needs to be measured for a range of Reynolds numbers. Even better curve fits can be produced by using a tailor-made curve fit.
5. When operating pipelines close to the 'smooth pipe'-line, as is often the case for high pressure gas pipelines, very high sensitivity to surface roughness must be expected. The friction factor may often be reduced very significantly by choosing appropriate coating or other

surface treatment.

6. Even moderate changes in relative roughness due to corrosion, wear, coating damage, and other unfavorable developments with time can have very significant effect on a pipeline's capacity. It is therefore desirable to estimate these effects during design, and to consider over-sizing according to uncertainties in those estimates.

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